Computations of Roll Motion in Waves Using a Fully Nonlinear Potential Flow Method

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Abstract. Optimization of modern hulls when moving in a seaway puts new demands on the computational methods used. Nonlinear effects become important for wave loads and added resistance in waves in presence of large motions. The purpose of this paper is to present a method which aims to fill the gap between RANSE methods and partly nonlinear panel methods. The method solves the fully nonlinear free surface time-domain potential flow problem including a hull undergoing rigid body motions. Nonlinearities under the hypothesis of potential flow are taken into account, i.e. higher and lower frequency components, hull shape above calm water line and interaction between incoming, radiated, diffracted, reflected and ship generated waves. The potential flow method alone cannot handle roll motion since roll is dominated by viscous effects. Two methods to include roll damping within the potential flow code are used: the first one obtains roll damping coefficients through inertial and geometric characteristics of the ship. The second one uses model test results. Numerical results using both methods are compared. The code has already been tested in head seas. In this paper, numerical simulations of roll decay and roll motion in beam sea are compared to model test results.

Keywords. Roll motion, Roll damping, Potential flow

1. Introduction

Nonlinear effects for ship motions in waves have become increasingly important for modern hulls designs, characterized by large bow flare and wide transom sterns. Nowadays, the most advanced numerical simulations methods used to predict ship motions are based on the solution of Navier-Stokes' equations; such methods are also capable of capturing nonlinearities due to viscous effects. The main drawback of these methods is the long computational time required, limiting the use for practical purposes. In order to solve this issue, a great part of the methods available today introduces the hypothesis of

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potential flow, where the fluid is assumed to be inviscid, incompressible and irrotational. Furthermore, many of these methods use a linear or partly nonlinear approach. Thanks to such further simplification, the computational effort required is reduced considerably. A review on the state-of-the-art of the codes available can be found in [1].
Under the hypothesis of potential flow, viscous effects are not taken into account. For longitudinal motions, such as pitch and heave, the error introduced can often be neglected. Roll motion, on the other hand, is normally highly influenced by viscosity: roll damping, among other terms, is dominated by viscous effects and under the hypothesis of potential flow, this motion cannot usually be evaluated properly. To solve this problem, artificial damping coefficients are added within the roll motion equation. Furthermore, roll motion is strongly nonlinear and characterized by large responses. When a linear approach is used to evaluate big amplitudes, the error introduced can be too high.
In this paper, Shipflow MOTIONS, a fully nonlinear 3-D unsteady potential flow method, is used. This code aims to fill the gap between viscous codes, where accuracy is achieved but with high computational effort, and linear or partially nonlinear codes, where computational effort is privileged over accuracy. The time required to run a simulation is kept low thanks to the hypothesis of potential flow but a good agreement between numerical simulations and model test experiments is achieved. In this code, higher and lower frequencies, the instantaneous wetted part of the hull and the interaction between waves and the hull are taken into account. Furthermore, in order to properly handle roll motion *i.e.* to take into account viscous effects, damping coefficients obtained in two different ways are used and numerical results compared. Results for longitudinal motions can be found in [2] and [3]. Here, the results of numerical simulations of roll decay and roll motion in beam sea with regular waves are presented.

2. Mathematical Model and Numerical Approach

Under the assumption of a homogeneous, inviscid, incompressible and irrotational fluid, there exists a scalar quantity called velocity potential ϕ, which is used to describe the motion of the fluid and satisfies the Laplace's equation:

$$\nabla^2 \phi = 0 \tag{1}$$

The kinematic and dynamic boundary conditions are imposed on the free surface:

$$\frac{D\mathbf{x}}{Dt} = \nabla \phi \tag{2}$$

where $\mathbf{x} = (x, y, z)$ is the position of a fluid particle on the free surface and

$$\frac{D\phi}{Dt} = -gz + \frac{1}{2}\nabla\phi \cdot \nabla\phi - \frac{p_a}{\rho} \tag{3}$$

where g is the gravitational acceleration, p_a is the atmospheric pressure and ρ is the fluid density. The material derivative is defined as

$$\frac{D}{Dt} \equiv \frac{\partial}{\partial t} + \nabla\phi \cdot \nabla \tag{4}$$

On the istantaneous wetted body surface an impermeability condition is imposed, taking into account rigid body motions:

$$\frac{\partial \phi}{\partial n} = \mathbf{n} \cdot (\mathbf{u} + \omega \times \mathbf{r}) \tag{5}$$

where $\mathbf{u}$ and ω are the translational and angular velocities, $\mathbf{r}$ is the position of the point where the condition is applied with respect to the center of rotation and $\mathbf{n}$ is the unit normal vector pointing into the fluid domain.
On the bottom of the fluid domain, an impermeability condition is imposed:

$$\frac{\partial \phi}{\partial n} = 0 \tag{6}$$

The mixed boundary value problem for Equation (1) is solved using a boundary element method, where sources are distributed on quadrilateral panels on the hull and on the free surface, see [4], and the evolution of the free surface is obtained by means of a *Mixed Euler-Lagrangian* (MEL) method, as described in [5], integrating equations (2) and (3) with a fourth order Adam-Bashforth-Moulton method.
In order to avoid unwanted reflections from domain boundaries as well as providing wave generation, a blending zone is introduced. This zone blends the computed solution with an analytically described solution known *a priori*, also functioning as wave generation.
Once the velocity potential is known on the hull, it can be used to obtain the pressure through the unsteady Bernoulli's formulation. Integrating the pressure on the instantaneous wetted hull, it is possible to obtain the forces and moments acting on the hull and thus to solve motion equations. Within the code, motion equations are solved taking into account also coupling terms. If we consider, for sake of simplicity, the *one Degree Of Freedom* (1DOF) roll motion equation and write it as it is computed by the code, we have:

$$I_{xx}\ddot{\varphi} = -\iint_{S_b} p\big(\mathbf{r} \times \mathbf{n}\big) \cdot \mathbf{i}\, dS \tag{7}$$

Thanks to the fact that the code is fully nonlinear and time dependent, in the right hand side of equation (7) all the components of roll motion equation (added mass, damping, restoring and forcing term) are simultaneously evaluated by the integral at each time step.
The BEM is based on the hypothesis of potential flow, *i.e.* viscosity is neglected. It must be noted that viscous effects mainly affect the damping term; in the code, the pressure is integrated on the instantaneous wetted surface and we can expect then a good evaluation of all terms except the viscous damping. In order to take into account the viscous damping then, coefficients have to be added within roll motion equation, which becomes, in 1DOF:

$$I_{xx}\ddot{\varphi} = -\iint_{S_b} p\big(\mathbf{r} \times \mathbf{n}\big) \cdot \mathbf{i}\, dS - B(\dot{\varphi}) \tag{8}$$

There are many ways to represent viscous damping, see for example [6]. The most used are:

$$
\begin{aligned}
B(\dot{\varphi}) &= B_l\dot{\varphi} + B_q\dot{\varphi}^2 + B_c\dot{\varphi}^3 \\
B(\dot{\varphi}) &= B_l\dot{\varphi} + B_q\dot{\varphi}^2 \\
B(\dot{\varphi}) &= B_l\dot{\varphi} + B_c\dot{\varphi}^3
\end{aligned} \tag{9}
$$

In this paper both the quadratic plus linear and the cubic plus linear formulations are used. The main problem is then to find a way to properly evaluate such coefficients. There are different methods available to do so, and these methods can be divided in two categories: the first one predicts such coefficients using geometrical and inertial characteristics of the ship, see [7]. The second group of methods analyzes model test results to obtain these coefficients, using either time series of roll decay or frequency domain curve of beam sea. Since the second group uses results which are specific for each ship, the prediction of the coefficients is expected to be more precise.

In this work, the coefficients are obtained with two methods, each one belonging to a different group. A formulation proposed by Watanabe and Inoue (W-I) and presented in [7], is used to predict the coefficients. Such formulation is based on a regression analysis and allows to obtain the quadratic term in Equation (9). Since adding the pure quadratic term alone proved not be enough, a linear term expressed as a percentage of the critical damping was added as well. The percentage was kept constant for all the hulls.

To evaluate the coefficients starting from model test results, a *Parameter Identification technique* (PIT) is used. This technique minimize the chi square error:

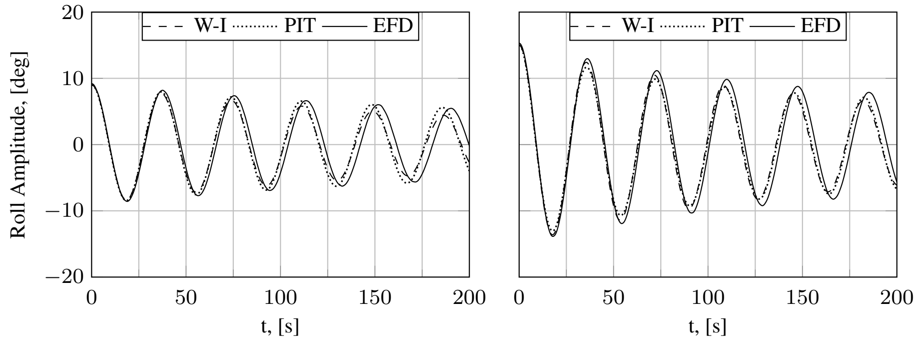
$$
\chi^2(\mathbf{p}) = \sum_{i=1}^{N_{data}} \left(\varphi(i) - \hat{\varphi}(i,\mathbf{p})\right)^2 \tag{10}
$$

finding $\mathbf{p}$, a set of parameter which gives the best fit between the input values φ and the numerically evaluated $\hat{\varphi}(\mathbf{p})$. In this work such parameters are the damping and restoring coefficients. It is possible to chose the damping model from equation (9) which best suits the specific need. For an insight of the method used here, see [8].

The results of roll decay simulations for a post-panamax container ship, the *Duisburg Test Case* (DTC), and beam sea results in regular waves for a *Series 60* (S60) hull are presented in the following section. The description of model tests can be found in [9] for the DTC and in [10] for the S60. To compare numerical simulations and model test results for roll decay, a qualitative study is done through the analysis of time domain curves. To have a better understanding of the quality of numerical simulations, roll frequency and linear equivalent damping are also compared between simulations and model tests: the frequency is evaluated considering the time between two consecutive peaks of the same sign and computed against the mean roll amplitude between suck peaks. To compare the linear equivalent damping we must recall that the linearized solution for roll decay is:

$$
\varphi(t) = \varphi_0 e^{-\mu_{eq}t}\cos(\omega_0 t) \tag{11}
$$

where φ_0 is the initial heeling angle, μ_{eq} is the equivalent linear damping and ω_0 is the natural frequency of roll motion. The damping between simulations and model test is compared fitting an exponential curve between to consecutive peaks of the same sign and plotting it against the mean roll amplitude between such peaks. Note that in this way, it is also possible to see how these two quantities (natural frequency and damping) vary with roll amplitude.



(a) *Initial heeling angle of* 9°.

(b) *Initial heeling angle of* 15°.

Figure 1. Time series of roll decay for the DTC.

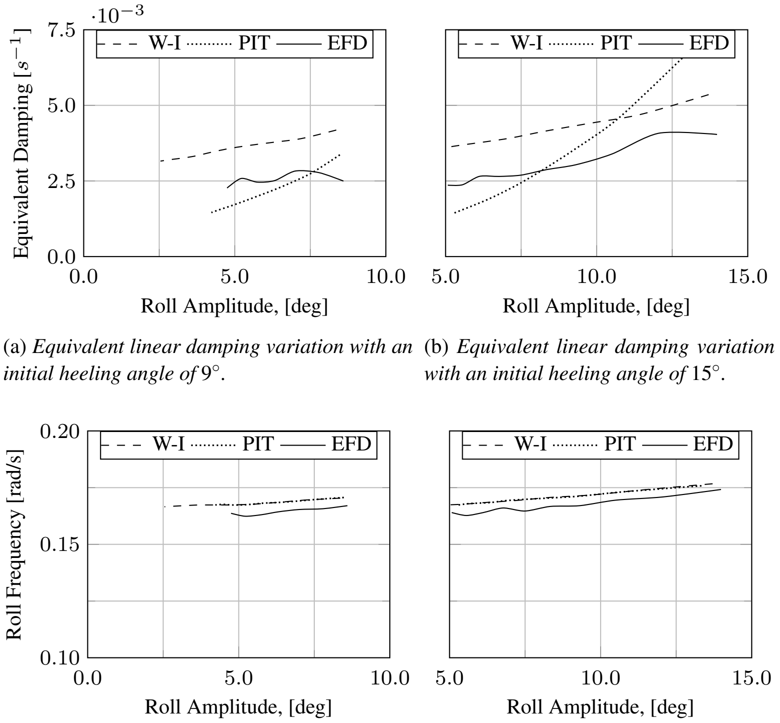
3. Results

For a description of the DTC ship properties and the set-up of the experiments see [9]. In order to find the minimum number of panels that can be used without losing too much accuracy, a grid dependency study was performed, comparing a coarse grid resolution and a medium resolution. The difference between these resolutions was noticed to be small: this allows to reduce the number of panels to be used in order to save some computational time.

Figure 1 shows the time series of roll decay with two different initial heeling angles. Both methods to obtain the damping coefficients were used: first the W-I formulation to get the quadratic term plus a linear term as a percentage of the critical damping and then the PIT. The PIT was used on the time domain response of model test results with both initial heeling angles. The viscous damping model used was a cubic plus linear. A comparison between the variation of equivalent linear damping and roll frequency with roll amplitude is presented in Figure 2. As can be seen, there is a general good agreement between simulations and model tests, both for the frequency and for the damping. As expected, the difference in the frequency is really small: since the restoring term is evaluated on the instantaneous wetted surface and viscous effects do not play a central role in such component, there is a good agreement between numerical simulations and model test. Regarding the damping, the mean value of the linear equivalent damping obtained using the PIT is closer to the model test value, compared with the one obtained using the W-I formulation. This was expected as well, being the PIT specifically related to the study case, compared to the generic W-I formulation.

After roll decay had been tested, numerical simulations of roll motion in beam sea with regular waves for a S60 were performed. Here, the ship was forced to roll with different wave frequencies, characterized by a constant wave steepness of $s_w = 1/100$, and the steady state response plotted against the wave forcing frequency. Ship characteristics and the set-up of experiments can be seen in [10]. Again, both formulations were used to obtain damping coefficients, but in this case the PIT was used to analyze the frequency response curve. The damping model adopted here with this method was pure linear and such value was kept constant for all wave frequencies.

For this hull, model test results of roll decay were not available. Numerical simulations of roll decay were performed to verify that roll natural frequency was evaluated properly.



(a) *Equivalent linear damping variation with an initial heeling angle of* $9°$.

(b) *Equivalent linear damping variation with an initial heeling angle of* $15°$.

(c) *Frequency variation with an initial heeling angle of* $9°$.

(d) *Frequency variation with an initial heeling angle of* $15°$.

Figure 2. Equivalent linear damping and frequency variation.

A good agreement between numerical simulations and model test results was found, as can be seen in Figure 3.

Beam sea results are presented in Figure 4, plotting the steady state of the response against the wave frequency and where the backbone curves, obtained for numerical simulations analyzing the roll frequency variation in roll decay tests, can also be seen. As can be clearly seen, the results where the PIT was used to get the damping coefficients are closer to experimental results. Furthermore, the bending of the peak of the response is in good agreement with model test with this technique. It must be noted in Figure 5 that, if the additional linear term was neglected from the damping term, added to have a better fit in roll decay tests, there is an improvement in the results. The root mean square errors, defined as $RMSE = \sqrt{\frac{\sum_{i=1}^{n}(y_i-\hat{y}_i)^2}{n}}$, where y_i is the experimental value and $\hat{y}_i$ the simulation result, can be seen in Table 1. The $RMSE_{cri}$ is obtained considering only the two biggest responses, intended as critical during the design since they represent the most dangerous situations.

Table 1. Root mean square errors for numerical simulations in beam sea.

error	Watanabe	Linear and Watanabe	PIT
RMSE	2.25	3.20	1.37
$RMSE_{cri}$	3.62	6.03	1.04

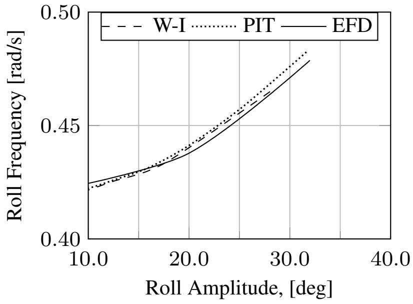
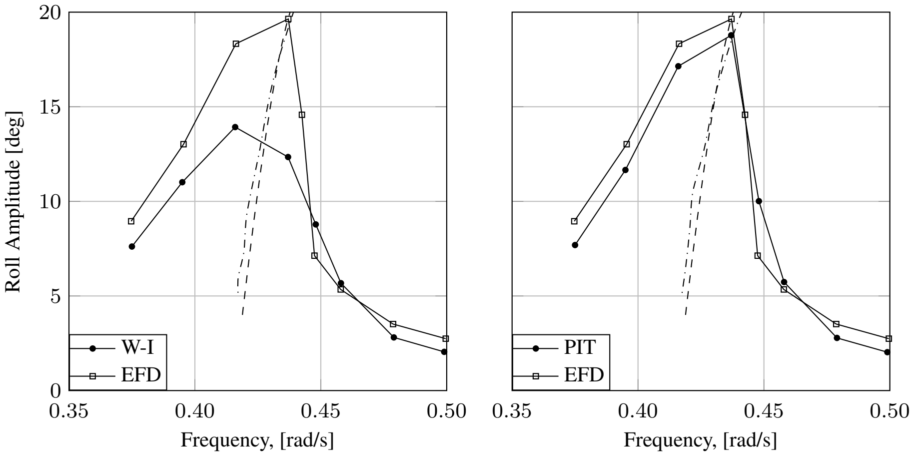


Figure 3. Comparison of roll frequency for the S60.



(a) *Beam sea response with W-I formulation.* (b) *Beam sea response with PIT formulation.*

Figure 4. Beam sea results with costant wave steepness $s_w = 1/100$.

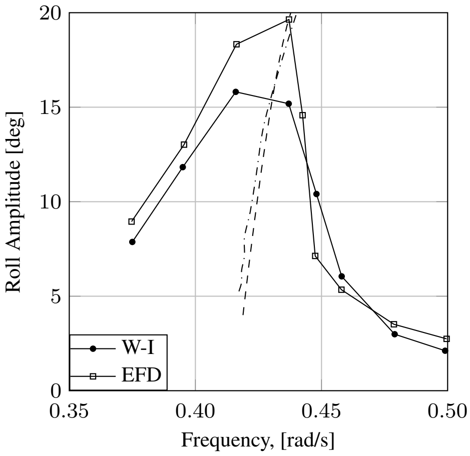


Figure 5. S60 beam sea response using only the quadratic term of W-I formulation.

4. Conclusions

A common approach to simulate ship motions is to introduce the hypothesis of potential flow. By doing so, a great improvement is achieved in terms of computational time, but on the other hand, there is the need to introduce viscous effects when dealing with roll motion. There are several methods to evaluate damping coefficients that are going to be added within roll motion equation. As seen in the previous section, regarding roll decay, a good agreement between model test and numerical simulations can be reached with both methods used. Obviously, in order to use the PIT, model test results must to be available; if they are not, the W-I formulation can be used without losing too much in terms of accuracy. However, if the W-I formulation is used to simulate roll motion in beam sea, the results are not that satisfactory. Even without the additional linear term in the damping coefficients, which proved to give a smaller difference between numerical simulations and model test, the error in the peak of resonance is around 25%. This could be due to that fact that roll motion in beam sea is a far more complex phenomenon, compared with roll decay. Furthermore, the formulation to get the quadratic coefficient with the W-I formulation is based on a regression analysis and could be therefore too simplistic.
To sum up, the code used proved to be able to handle roll motion. The accuracy of the results is mainly correlated to the one used to evaluate damping coefficients. Furthermore, thanks to the hypothesis of potential flow, the computational time needed to perform the simulations is kept small and makes the code usable also for practical purposes.
It is clear that the test process is not over yet: other simulations of roll motion in beam sea have to be carried out as well as some studies on parametric rolling. Furthermore, some other way to predict damping coefficients when model tests are not available could be an interesting point to investigate.

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