Evaluation of extreme wave loads for slender tubular structures

Francesco MAURO [a,b,1] and Marco MONACOLLI [a]
[a] *Department of Engineering and Architecture, University of Trieste, Via Valerio 10, 34100 Trieste, Italy*
[b] *Faculty of Engineering, University of Rijeka,Vukovarska Ulica 58, 51000 Rijeka, Croatia*

Abstract. To design particular Offshore Vessels appendages like stingers, it is common practice to search extreme values of wave induced loads. The standard methods applied are performing the analysis by means of a Weibull distribution. The necessity of offshore industry to operate with severe sea state and the complexity of the considered geometry can be source of evident non-linearities in the peaks distribution of the exciting force. In the specific, the adoption of a standard Weibull approach is not indicated for accurately predict the extreme load value. The adoption of more accurate distributions suitable to capture peaks non-linearity will ensure to overcome or capture possible multi-modal behaviours of the considered population. Such techniques can be applied since early design stage also to calculation results. In the present work a methodology is applied to calculation results for a stinger geometry, where Morison theory is applied to evaluate wave loads considering shield effects between the single elements.

Keywords. Extreme loads, Extreme Value Theory, Morison equations

1. Introduction

Besides experimental model tests, numerical simulations are a useful support for Offshore designers in such a way to determine the extreme values of loads and motions. In particular, for a correct dimensioning of fixed structures and ships appendages, the determination of wave induced loads in harsh condition is required. Both in case of a model test or a simulation, time series of the measured/calculated loads have to be analysed by extracting in an appropriate way the peaks.
According to extreme value theory [1] [2], the Generalised Extreme Value Distribution (GED) should be used when all the peaks are considered. Once only the peaks above a certain threshold are selected for the analysis, then Generalised Pareto Distribution (GPD) should be adopted [3]. In ship and offshore design it is common practice to analyse the peaks as suggested by ITTC [4] by means of a two or three parameters Weibull distribution. However, once complex structure are investigated in rough sea, than non-linear behaviours can be identified in the peaks distributions [5]. In such a case the Weibull distribution in its standard form will not estimate extreme values satisfactory [6],

[1] Corresponding author: fmauro@units.it

but multi-modal distributions [7] or GPD distribution [8] should be adopted, according to the selected peaks extraction method.
In the particular case of relative slender tubular structures, preliminary calculations can be done modelling wave forces according to Morison equation [9]. In particular, once complex tubular structures have to be analysed, the model suitable for vertical and horizontal cylinders can be extended for generally inclined ones [10], in such a way to evaluate all the possible sections of a tubular structure [11]. Also in case of calculations considering non-linearities, the peaks distribution presents a multi-modal behaviour. In the present work the specific case of a stinger is presented, where calculations have been executed with a self-developed program capable to evaluate the wave induced forces on general slender structures [12], considering also shield effects between adjacent piles.
The results obtained on the selected structure are here presented for one of the tested solution, being representative for all the simulations, and the differences between traditional peaks analysis method and proposed enhanced procedure are shown, highlighting the possible mistakes that a wrong extreme distribution modelling can generate in structure dimensioning.

2. Wave forces determination

The determination of total wave forces acting on a complex structure are here determined by means of a calculation method based on Morison equation. A tubular structure like a stinger is composed by a certain number of cylindrical piles disposed with different incidence angles with respect to the incoming flow. For such a reason the standard Morison equation, referring to vertical and horizontal cylinders, should be written in a more general form, considering the effective cylinder inclination:

$$\underline{\mathrm{f}}_N = C_M \rho \frac{\pi}{4} D^2 \underline{\dot{\mathrm{w}}} + \frac{1}{2} C_D \rho D \underline{\mathrm{w}} |\underline{\mathrm{w}}| \tag{1}$$

where $\underline{\mathrm{f}}_N$ is the normal force per unit length, ρ is the water mass density, D the cylinder diameter and C_M and C_D are the inertia and drag force coefficients respectively. The equation refers to acceleration and velocity vectors ($\underline{\dot{\mathrm{w}}}$, $\underline{\mathrm{w}}$) normal to the cylindrical pile, that can be expressed in the incoming flow reference system according to the following transformations:

$$\underline{\mathrm{w}} = (\underline{\mathrm{u}} \cdot \underline{\mathrm{n}})\, \underline{\mathrm{n}} \tag{2}$$

$$\underline{\dot{\mathrm{w}}} = (\underline{\dot{\mathrm{u}}} \cdot \underline{\mathrm{n}})\, \underline{\mathrm{n}} \tag{3}$$

being $\underline{\mathrm{u}}$ and $\underline{\dot{\mathrm{u}}}$ are the water particle velocity and acceleration and $\underline{\mathrm{n}}$ is the unit vector normal to the pile and in the plane formed by the pile axis and $\underline{\mathrm{u}}$. The resulting forces breakdown on a general cylindrical pile inclined of an angle α with respect to water particle velocity is presented in Figure 1.
As mentioned, structures as a stingers are composed by multiple cylindrical piles, so equation 1 should be applied for each pile of the structure, considering that each part is facing a different load, due to the position of the piles with respect to the incoming wave system. By considering a simple superposition between the forces evaluated per each single pile an overestimation in the total wave force will be determined, because each

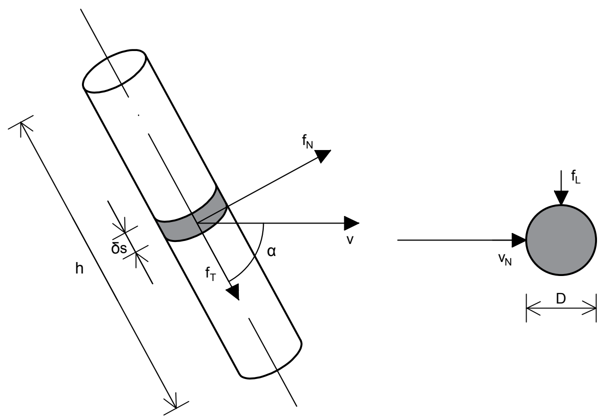


Figure 1. Forces scheme on a generally inclined cylindrical pile.

pile is considered invested by a uniform velocity field.
In a structure like a stinger the piles are close to each other, so it is reasonable to suppose that only the piles directly facing the flow will be subjected to a total force compliant with equation 1. The other piles will be in the wake of the front ones, being subjected to a shield effect [13], which is reducing the incoming speed on the piles' portions inside the wake of the forward ones, according to the relative positions between the piles adopting Schlichtling formulation extended for an Airy wave potential:

$$u_{TOT}(\xi,\eta,\zeta,t) = u_0(\xi,\eta,\zeta,t) - u_w(\xi,\eta,\zeta,t) \tag{4}$$

$$u_w(\xi,\eta,\zeta,t) \quad = k_1 u_0 \sqrt{\frac{C_D D}{\xi_S}} e^{-0.693\left(\frac{\eta}{b}\right)^2} \tag{5}$$

where u_0 is the incoming wave velocity and u_w is the wake velocity reduction, with k_1 a constant equal to 1.0 and b, ξ_S defined as follows:

$$b = k_2\sqrt{C_D D \xi_S} \tag{6}$$

$$\xi_S = \xi + \frac{4D}{C_D} \tag{7}$$

being k_2 a constant value set at 0.25. The reference scheme of the wake calculation is reported in Figure 2, it must be noted that this formulation is considering also a bi-dimensional approximation of the wake.
The described formulations are valid for regular waves. However to properly analyse extreme value theory, the irregular waves should be considered. Considering an irregular sea state modelling by means of a wave amplitude spectrum S_ζ, then the single amplitudes of multiple regular waves can be determined per each desired frequency band $\delta\omega$. In such a way the irregular wave system can be described as:

$$\zeta(t) = \sum_{i=1}^{N} \zeta_{0_i} \cos(k_i \xi - \omega_i t + \Phi_i) \tag{8}$$

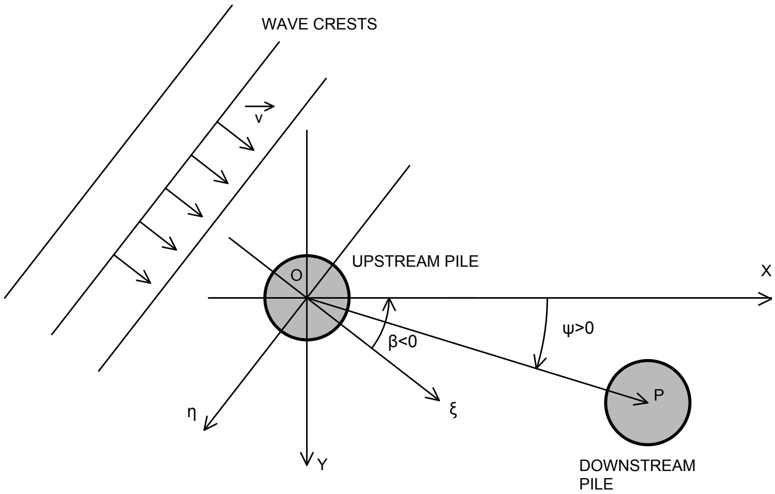


Figure 2. Reference system for wake determination on adjacent piles.

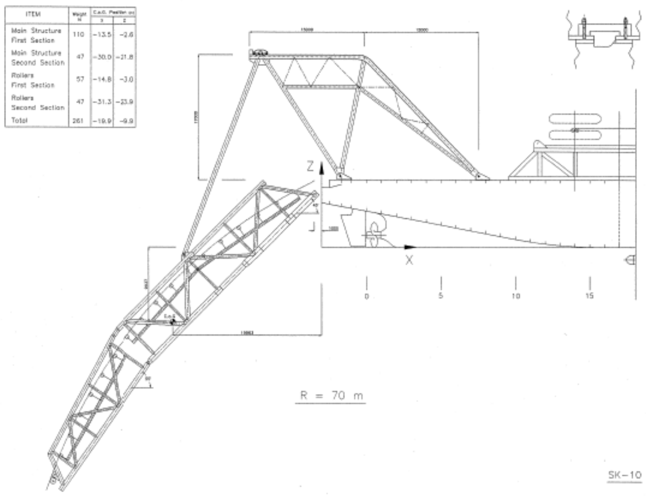


Figure 3. Overview of stinger geometry in working position used for the calculations.

where the phase Φ is generated with a random based process.
Adopting the above described calculation scheme, dedicated simulations have been carried out on an existing stinger geometry (Fig. 3), tested in a towing tank and already used to validate the force determination procedure [12]. Calculations have been carried out both in irregular seas. Through the validation study, several conditions in therms of sea state and incoming wave directions have been tested. Here, only one specific case is reported as example, considering a Bretschneider wave spectrum with a $H_{1/3}$ of 3.0 m and a T_z of 6.0 s with an incoming direction of 90 deg. The obtained forces records for transversal and vertical force are reported in Figure 4. For the presented condition

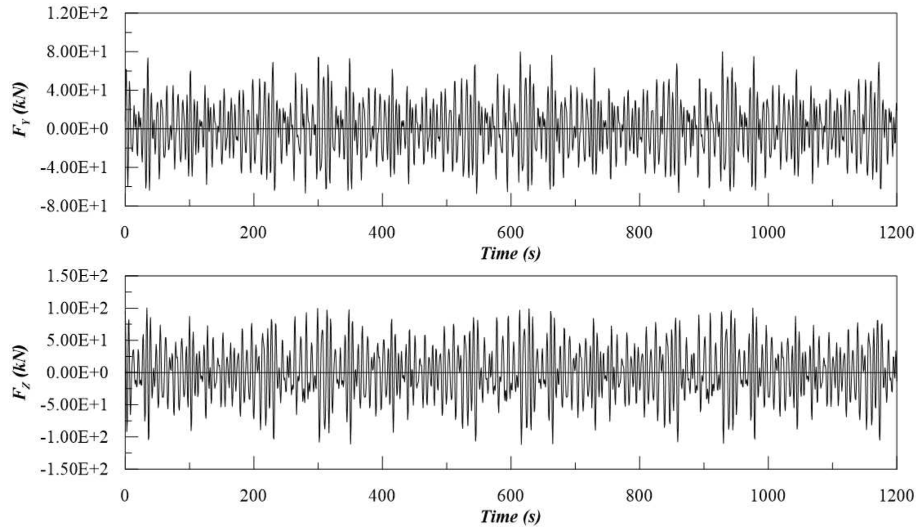


Figure 4. Wave forces in y (upper) and z (lower) direction for α=90 deg, $H_{1/3}$= 3.0 m and T_z=6.0 s.

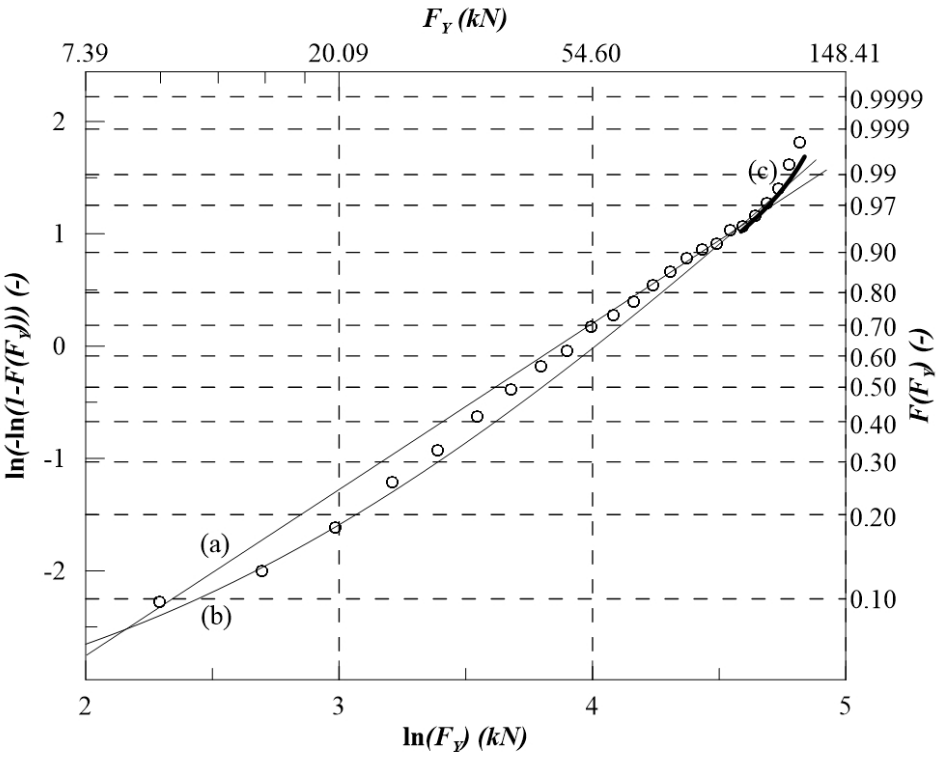
the longitudinal force is negligible with respect to the other two components, so is not reported in the analysis.

3. Extreme values determination

Once the forces have been evaluated and a time record is available, it is possible to start with the extreme value analysis, beginning with the peaks extraction from the obtained records. Peaks extraction process is of primary importance for an appropriate extreme value determination. In fact, according to the adopted extraction procedure, different distributions should be used for the extremes prediction.
Basically, two are the main possibility to extract peaks from a time series: the *Block Maxima* method and the *Peak Over Threshold* (POT) method. The first one consist in extract all the peaks of the time series while the second one is considering only the peaks that are above a certain determined threshold level. According to extreme value theory [2][3], once Block Maxima is selected, then the extremes, applying *Fisher-Tippet-Gnedenko* theorem, should be modelled with GEV distribution. The particular GEV-form of the Weibull distribution is usually adopted, considering a probability density function (PDF) written in the following form:

$$p(x) = \frac{\beta}{\eta}\left(\frac{x-\gamma}{\eta}\right)^{\beta-1} e^{-\left(\frac{x-\gamma}{\eta}\right)^{\beta}} \tag{9}$$

where $\beta \in (0,+\infty)$ is the shape parameter, $\eta \in (0,+\infty)$ is the scale parameter and $\gamma \in (-\infty,+\infty)$ is the location parameter. In equation 9, Weibull distribution is expressed in the more general three parameters form, once γ is set to 0, then the distribution became a two parameter Weibull witch is commonly used in Naval Architecture and Offshore Engineering for the extreme values determination.



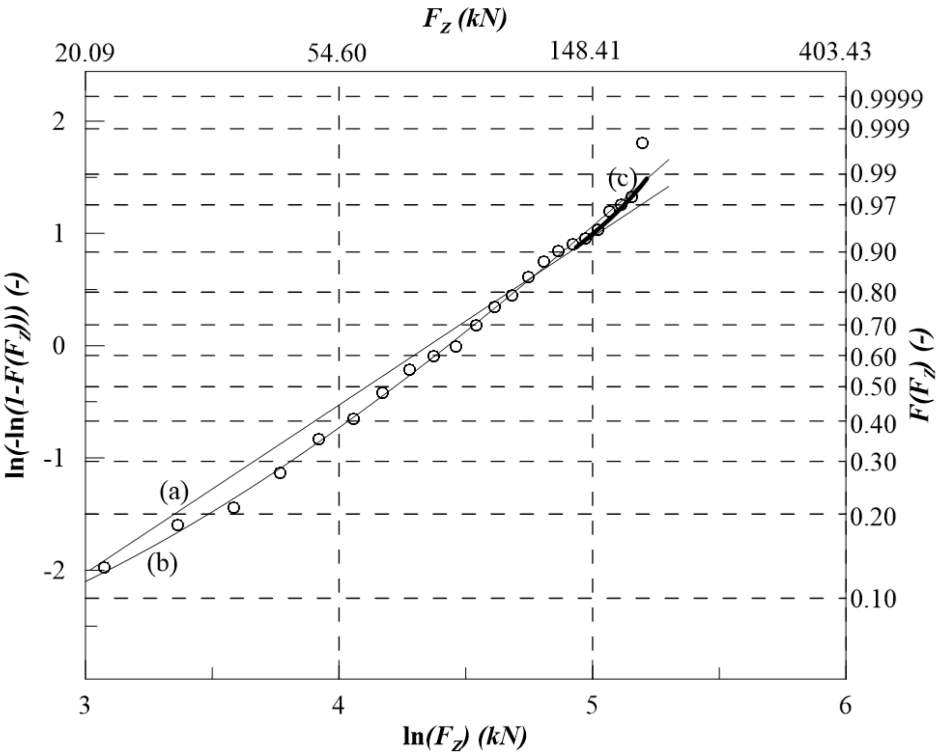


Figure 5. Analysis on the Weibull plot according to (a) 2 par. Weibull, (b) 3 par. Weibull and (c) GPD.

Table 1. Extreme values of F_Y and F_Z in kN.

	F_Y			F_Z		
	3.0%	1.0%	0.1%	3.0%	1.0%	0.1%
2 par. Weibull	111.17	133.75	175.80	179.62	215.47	282.47
3 per. Weibull	110.48	127.23	149.78	163.56	187.72	229.69
GPD	109.94	121.51	137.81	152.64	161.87	200.34

By selecting POT method, then *Pickands-Balkema-de Haan* theorem should be applied and consequently the extremes have to be modelled with a GPD distribution, having the following PDF:

$$p(x) = \frac{1}{\eta}\left(1+\beta\frac{x-\gamma}{\eta}\right)^{-\frac{1}{\beta+1}} \tag{10}$$

with $\beta \in (-\infty,+\infty)$, $\eta \in (0,+\infty)$ and $\gamma \in (-\infty,+\infty)$. The function is defined for $x > \gamma$ when $\beta > 0$, otherwise for $\gamma < x < \gamma - \frac{\eta}{\beta}$ when $\beta < 0$. Adopting a GPD to fit a certain distribution, presupposes that a suitable high threshold value can be found in such a way that the approximation suggested by Balkema and de Haan theorem is good and above which a sufficient number of data is present to ensure an accurate estimation of the unknown parameters. In the present study the threshold has been selected according the sample mean excess plot method as described in [8].
The calculations on the presented structure are not presenting clearly the feature of multimodal behaviours like the one presented in [7], so the analysis has been restricted in a comparison between standard Weibull distribution and GPD distribution. Generally for the Offshore structure dimensioning, the values of interest are referring to the events associated to the occurrence probability of $p = 3.0\%$, 1.0% and 0.1%. To find these values, use should be made of the *quantile* (inverse cumulative distribution) of the selected distribution. For the analysed distributions, the quantiles have the following form:

$$Q(p,\eta,\beta) = \eta\left(-\ln(1-p)\right)^{-\frac{1}{\beta}} \tag{11}$$

$$Q(p,\eta,\beta,\gamma) = \gamma+\eta\left(-\ln(1-p)\right)^{-\frac{1}{\beta}} \tag{12}$$

$$Q(p,\eta,\beta,\gamma) = \gamma+\frac{\beta}{\eta}\left(1-\left(\frac{n}{N_\gamma}(1-p)\right)^{\beta}\right) \tag{13}$$

which are representative of two, three parameters Weibull distribution and GPD distribution respectively. In particular, for the GPD, n represents the total number of samples of the record and N_γ the number of samples above the selected threshold γ. For the considered records, a threshold of 97 kN has been considered for F_y and 134 kN for F_z. The obtained data from the extreme value analysis are presented in Figure 5 and Table 1. As it can be seen the standard prediction methods are not able to properly fit the peaks distribution in the so called *tale*, resulting in an overestimation of the extremes of about 30% in case of the two parameter Weibull and about 20% for the three parameter case. For such a reason it is advisable to model extremes with a correct tale modelling, as suggested by the GPD approach.

4. Conclusions

On the selected stinger structures, dedicated non-linear calculations have been carried out to determine wave induced Forces by means of an extended Morison formulation. On the obtained records, standard Weibull analysis and an enhanced method based on GPD distribution have been carried out to predict extreme values of the evaluated forces. The obtained results highlights that by using standard 2 or 3 parameters Weibull distribution the extreme values can be overestimated, suggesting to apply the enhanced method based on GPD in order to improve the structure dimensioning during the design process.

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